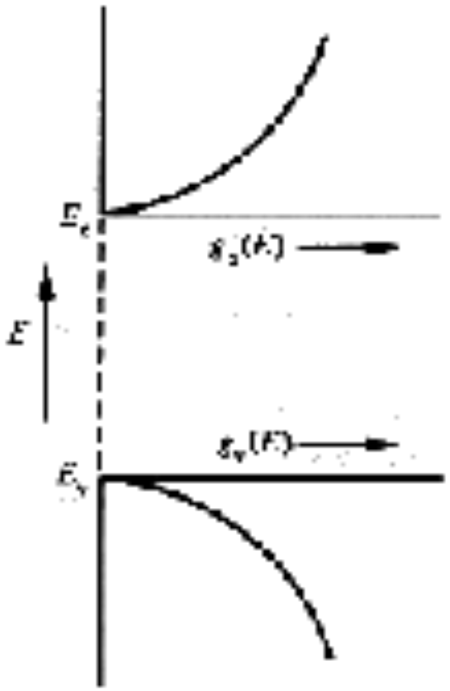
Density of States
and
Fermi Energy Concepts

How do Electrons and Holes Populate the Bands?

- Density of States Concept



General energy dependence of $g_c(E)$ and $g_v(E)$ near the band edges.

$$g_c(E)\,dE$$

The number of conduction band states/cm^3 lying in the energy range between E and $E + dE$ (if $E \geq E_c$).

$$g_v(E)\,dE$$

The number of valence band states/cm^3 lying in the energy range between E and $E + dE$ (if $E \leq E_v$).

How do Electrons and Holes Populate the Bands?

- Density of States Concept

  Quantum Mechanics tells us that the number of available states in a cm^3 per unit of energy, the density of states, is given by:

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3}, \quad E \geq E_c$$

Density of States in Conduction Band

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3}, \quad E \leq E_v$$

Density of States in Valence Band

$$unit \equiv \left(\frac{Number\ of\ States}{cm^3} \right) \Big/ eV$$

How do electrons and holes populate the bands?

- **Probability of Occupation** (*Fermi Function*) Concept
  - Now that we know the number of available states at each energy, then how do the electrons occupy these states?
  - We need to know how the electrons are "distributed in energy".
  - Again, Quantum Mechanics tells us that the electrons follow the "*Fermi-distribution function*".

$$f(E) = \frac{1}{1 + e^{(E - E_f)/kT}}$$

E_f ≡ Fermi energy (average energy in the crystal)
k ≡ Boltzmann constant (k=8.617×10^{-5}eV/K)
T ≡Temperature in Kelvin (K)

- *f(E)* is the probability that a state at energy E is *occupied.*
- *1-f(E)* is the probability that a state at energy E is *unoccupied.*

- Fermi function applies only under equilibrium conditions, however, is universal in the sense that it applies with all materials-insulators, semiconductors, and metals.

How do electrons and holes populate the bands?

❑ Fermi-Dirac Distribution

- Applies to the particles that obey the exclusion principle. (*Fermions*)
- Fermi-Dirac Function: The probability that a particular state E is occupied for a systems of Fermions.

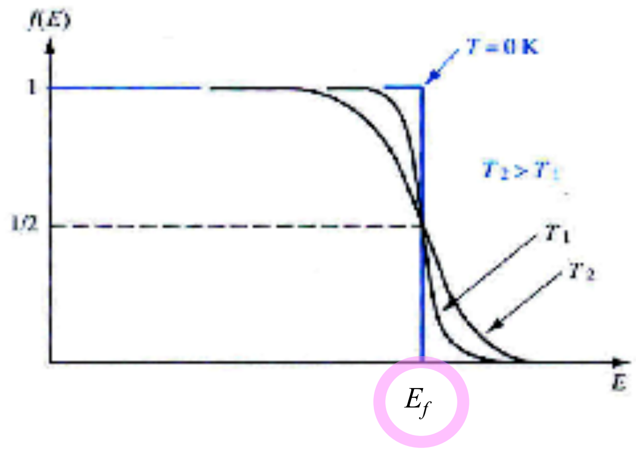
$$f(E) = \frac{1}{1 + \exp\left[(E - E_F)/kT\right]}$$

- E_F: Fermi Energy (Fermi Level)
- Some properties of F-D distribution function:

$$f(E) \le 1$$

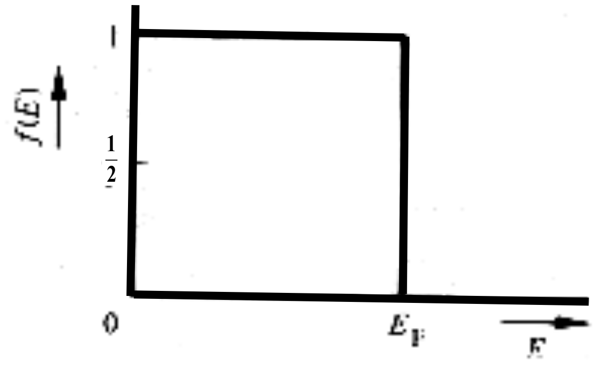
$$f(E = E_F) \le \frac{1}{2}$$

$$f(E_F + \delta E) = 1 - f(E_F - \delta E) = \frac{1}{1 + \exp(\delta E/kT)}$$

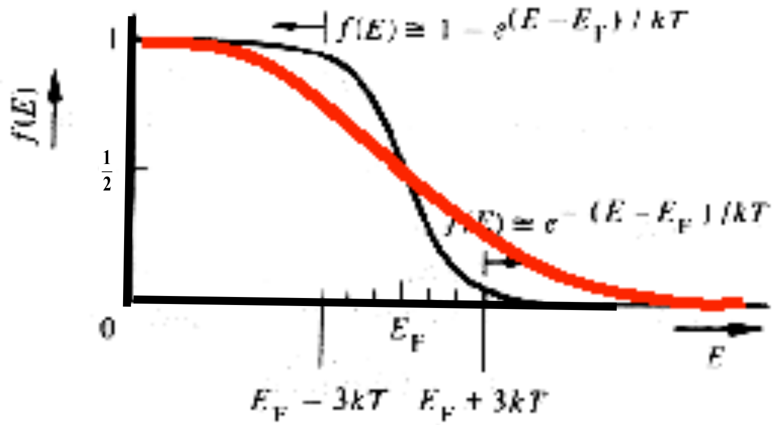


How do electrons and holes populate the bands?

- Probability of Occupation (*Fermi function*) Concept



(a) T → 0 K



(b) T > 0 K

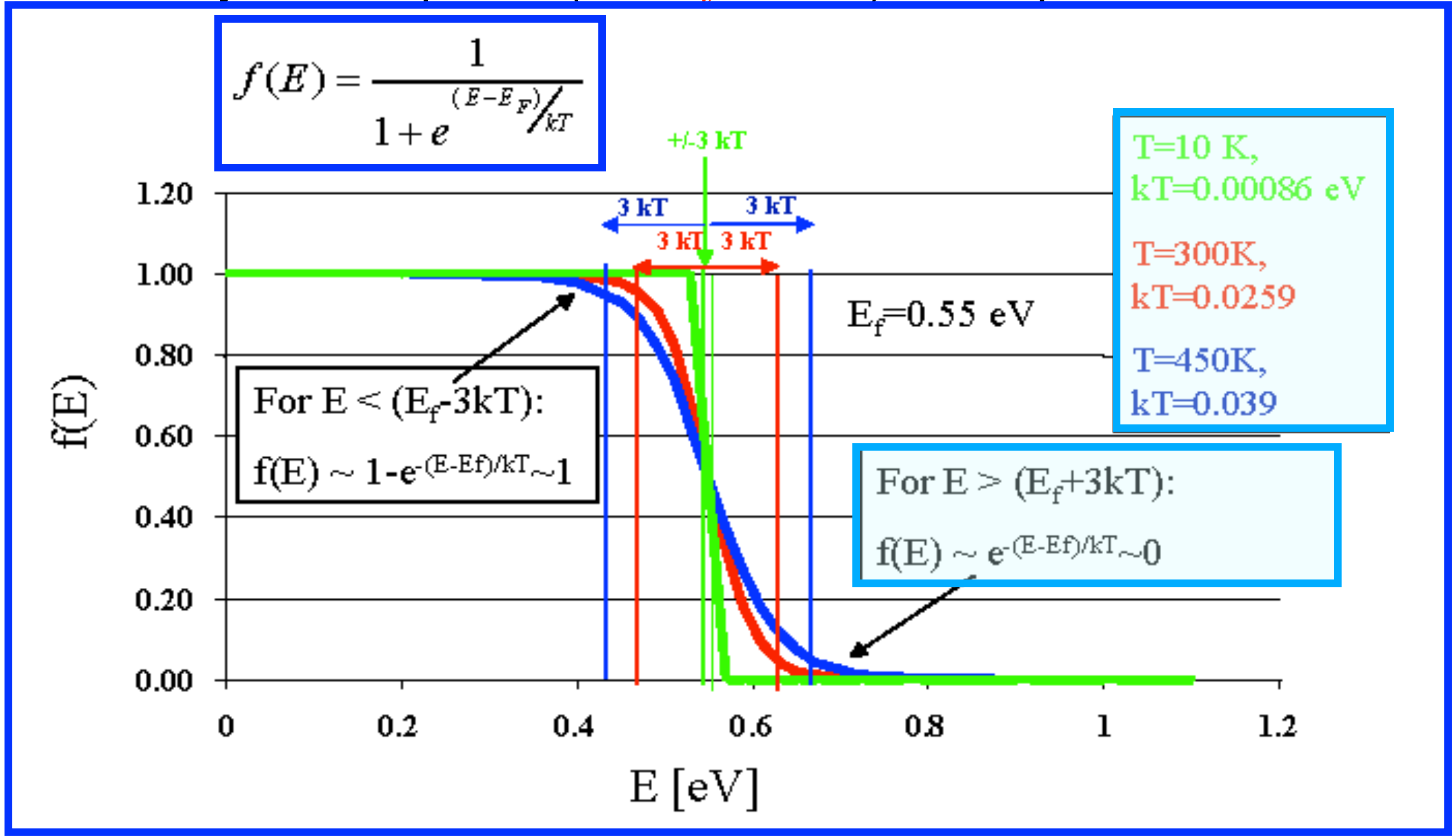
$$f(E) = \frac{1}{1 + e^{(E - E_f)/kT}}$$

$kT = 0.0259eV$ @300K

- At T=0K, occupancy is "digital": No occupation of states above E_f and complete occupation of states below E_f.
- At T>0K, occupation probability is reduced with increasing energy. $f(E=E_f) = 1/2$ regardless of temperature.
- At higher temperatures, higher energy states can be occupied, leaving more lower energy states unoccupied $[1 - f(E_f)]$.

How do electrons and holes populate the bands?

- Probability of Occupation (*Fermi function*) Concept



- If $E \geq E_f + 3kT$ → $e^{(E-E_f)/kT} \gg 1$ and $f(E) \approx e^{-(E-E_f)/kT}$
- Consequently, above $E_f + 3kT$ the Fermi function or filled-state probability decays exponentially to zero with increasing energy.

How do electrons and holes populate the bands?

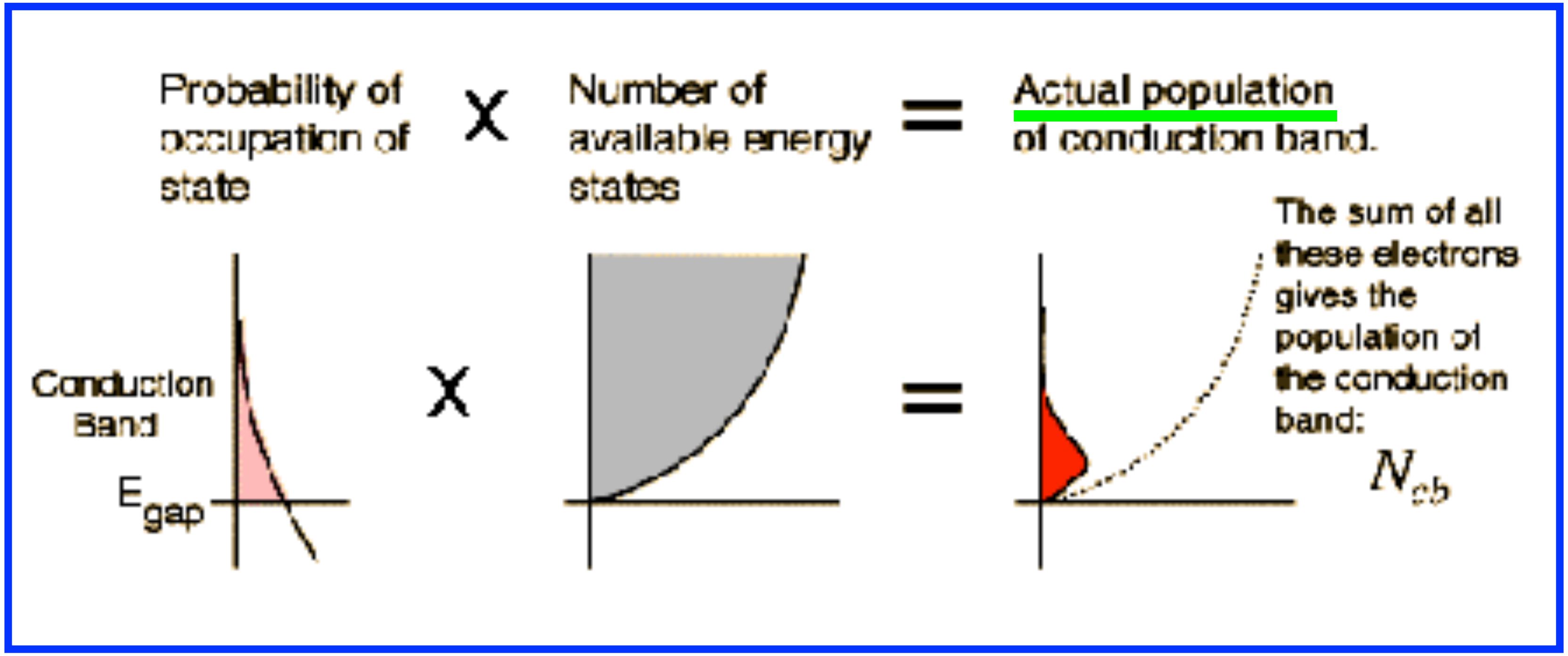
- Probability of Occupation Concept

The density of electrons (or holes) occupying the states in energy between E and $E + dE$ is:

$g_c(E)f(E)dE$	Electrons/cm^3 in the conduction band between E and $E + dE$ (if $E \geq E_c$).
$g_v(E)f(E)dE$	Holes/cm^3 in the conduction band between E and $E + dE$ (if $E \leq E_v$).
0	*Otherwise*

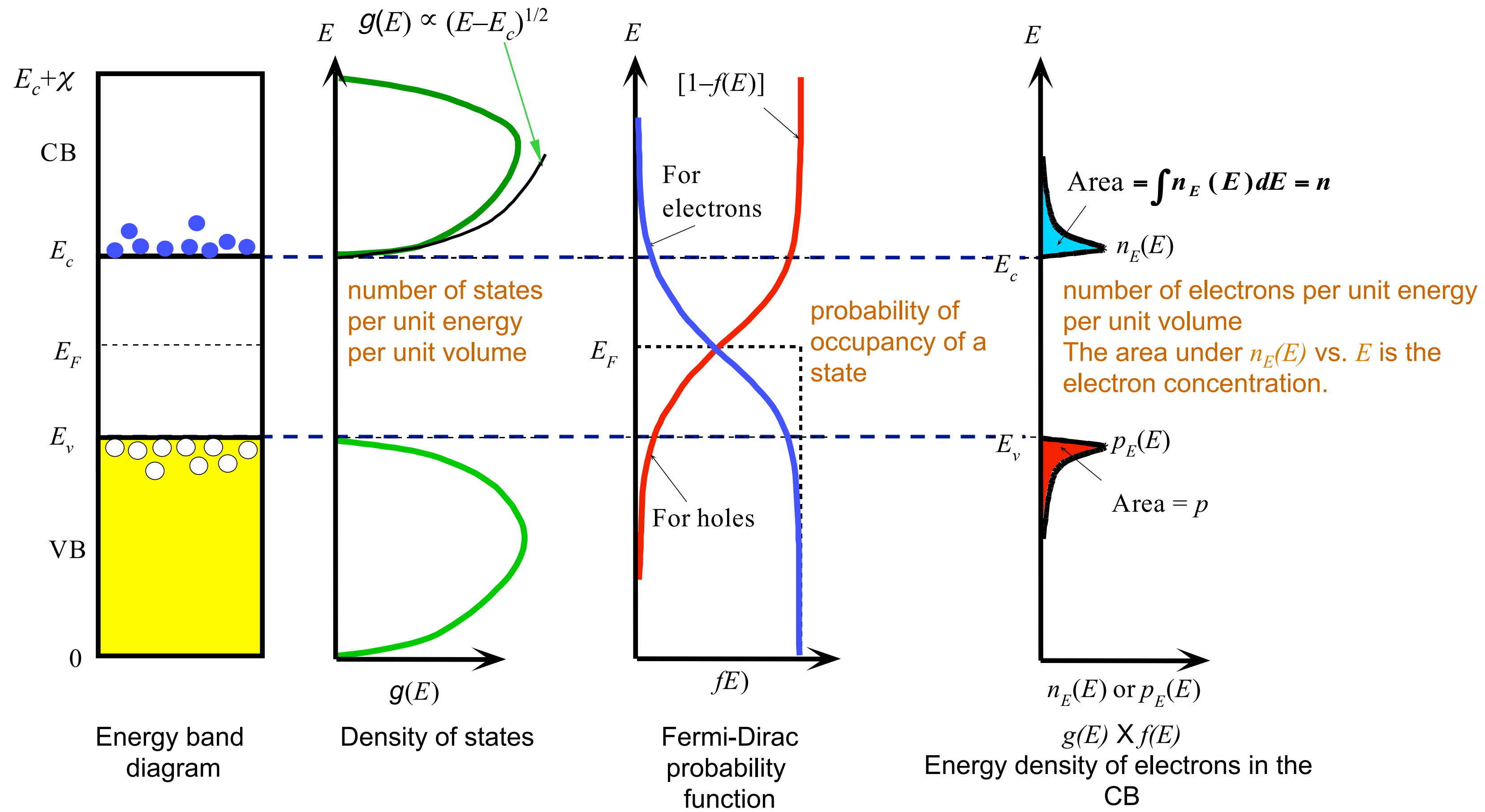
How do electrons and holes populate the bands?

- Probability of Occupation Concept



How do electrons and holes populate the bands?

- Typical band structures of Semiconductor

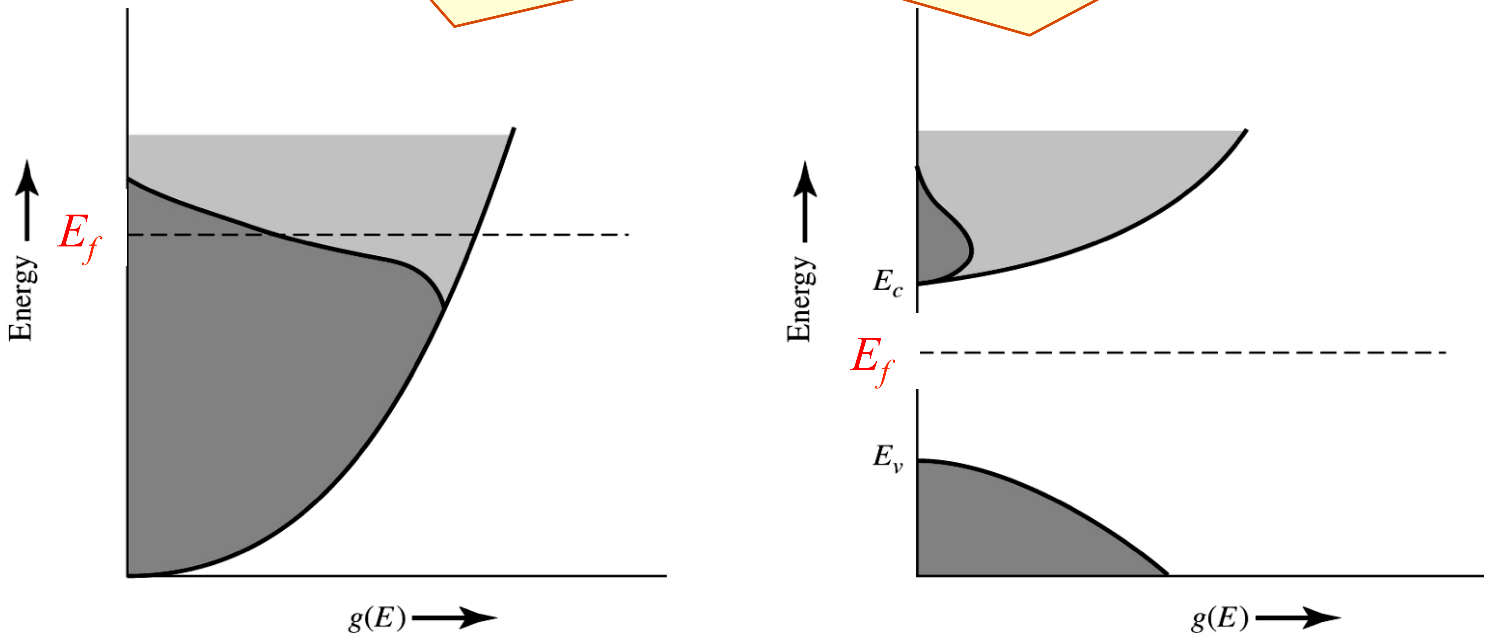


Metals vs. Semiconductors

- Allowed electronic-energy states $g(E)$

Fermi level E_f immersed in the continuum of allowed states.

The Fermi level E_f is at an intermediate energy between that of the conduction band edge and that of the valence band edge.



Metal

Semiconductor

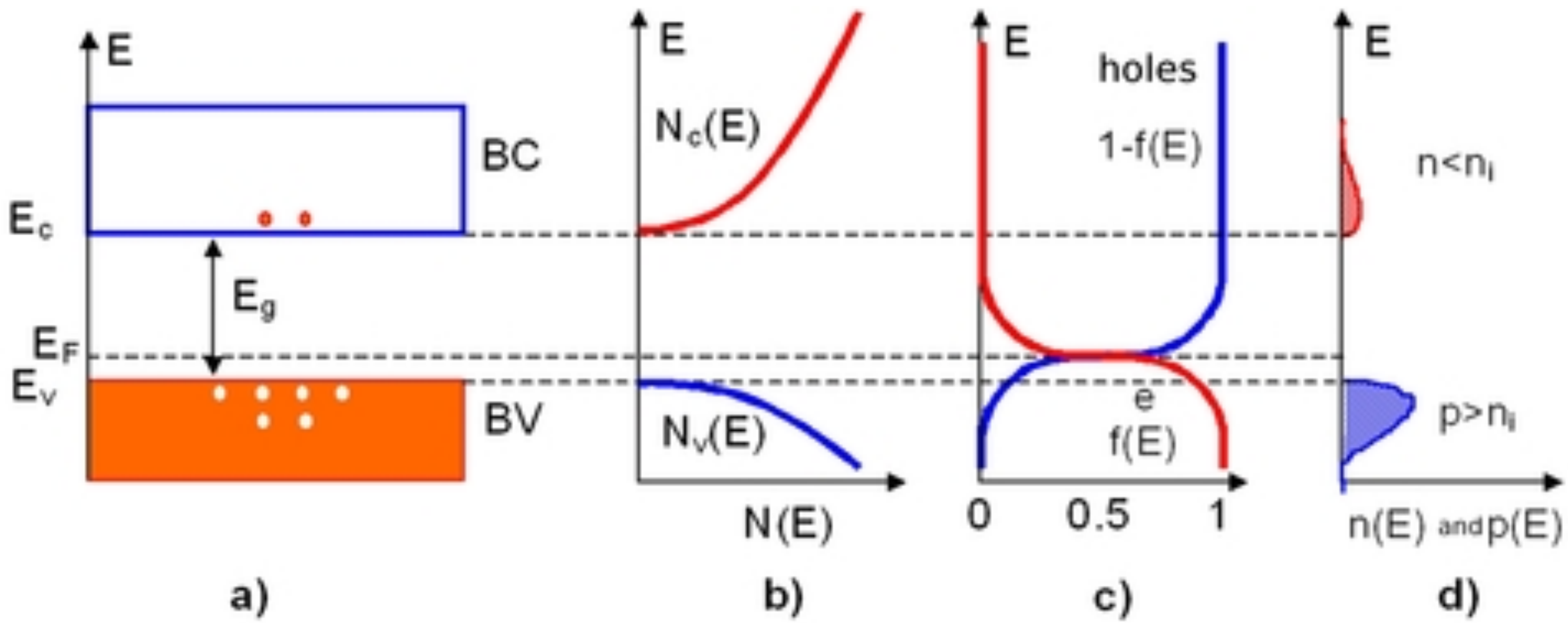
How do electrons and holes populate the bands?

Fermi function and Carrier Concentration

- Note that although the Fermi function has a finite value in the gap, there is no electron population at those energies. (that's what you mean by a gap)
- The population depends upon the product of the Fermi function and the electron density of states. So in the gap there are no electrons because the density of states is zero.
- In the conduction band at 0K, there are no electrons even though there are plenty of available states, but the Fermi function is zero.
- At high temperatures, both the density of states and the Fermi function have finite values in the conduction band, so there is a finite conducting population.

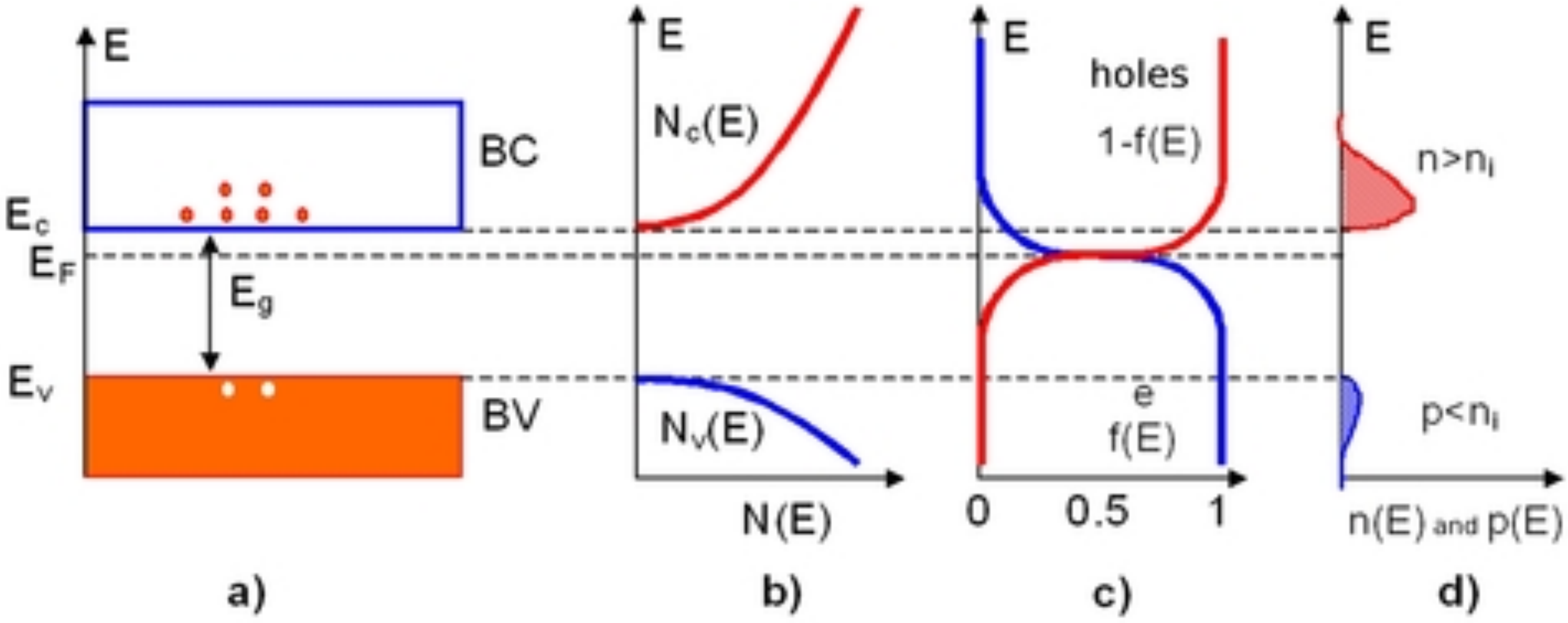
How do electrons and holes populate the bands?

- Energy Band Occupation in p-type semiconductor



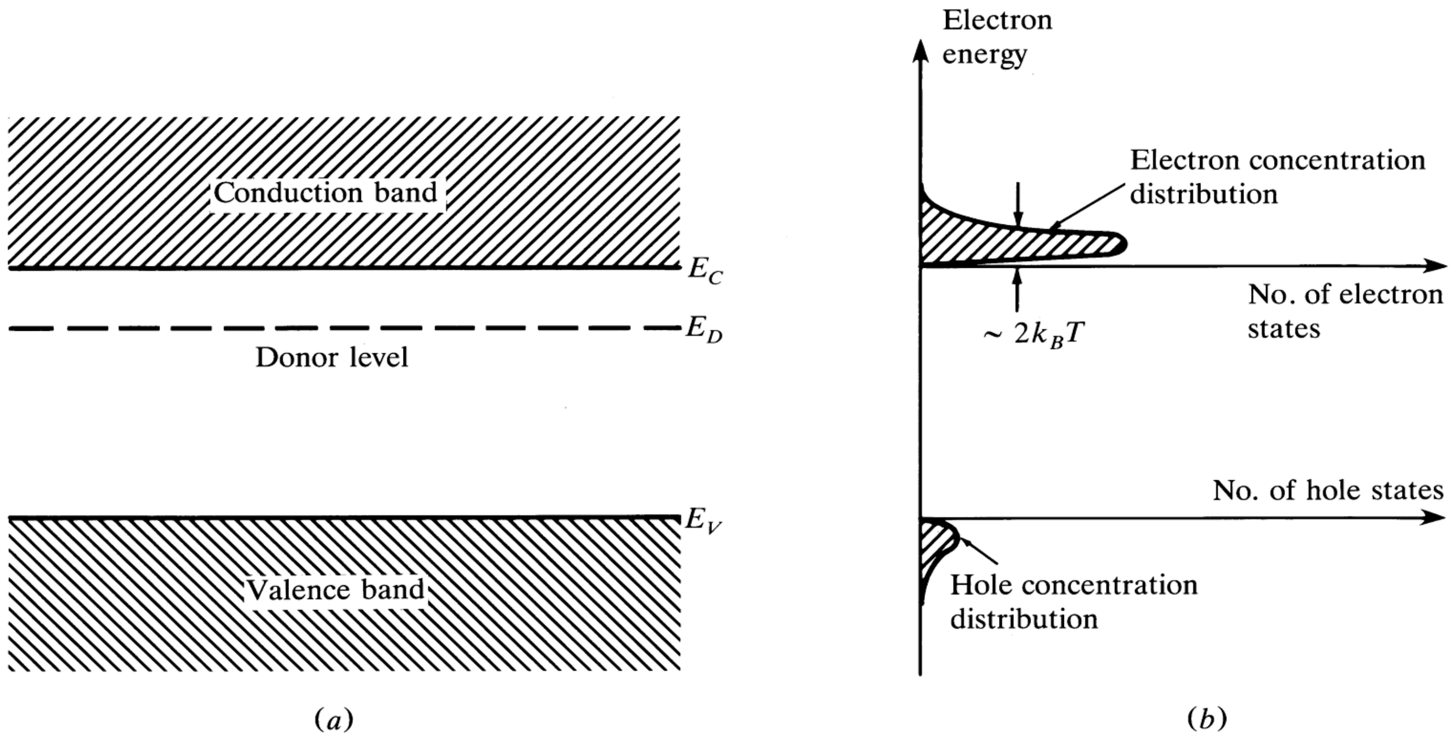
How do electrons and holes populate the bands?

- Energy Band Occupation in n-type semiconductor



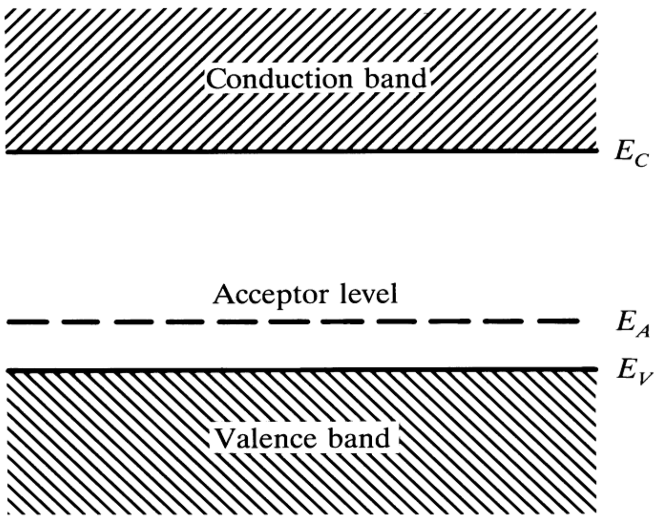
How do electrons and holes populate the bands?

- n-type material

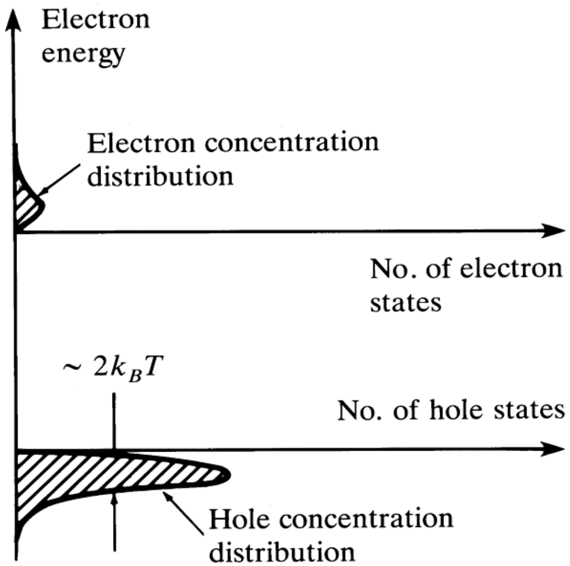


How do electrons and holes populate the bands?

- p-type material



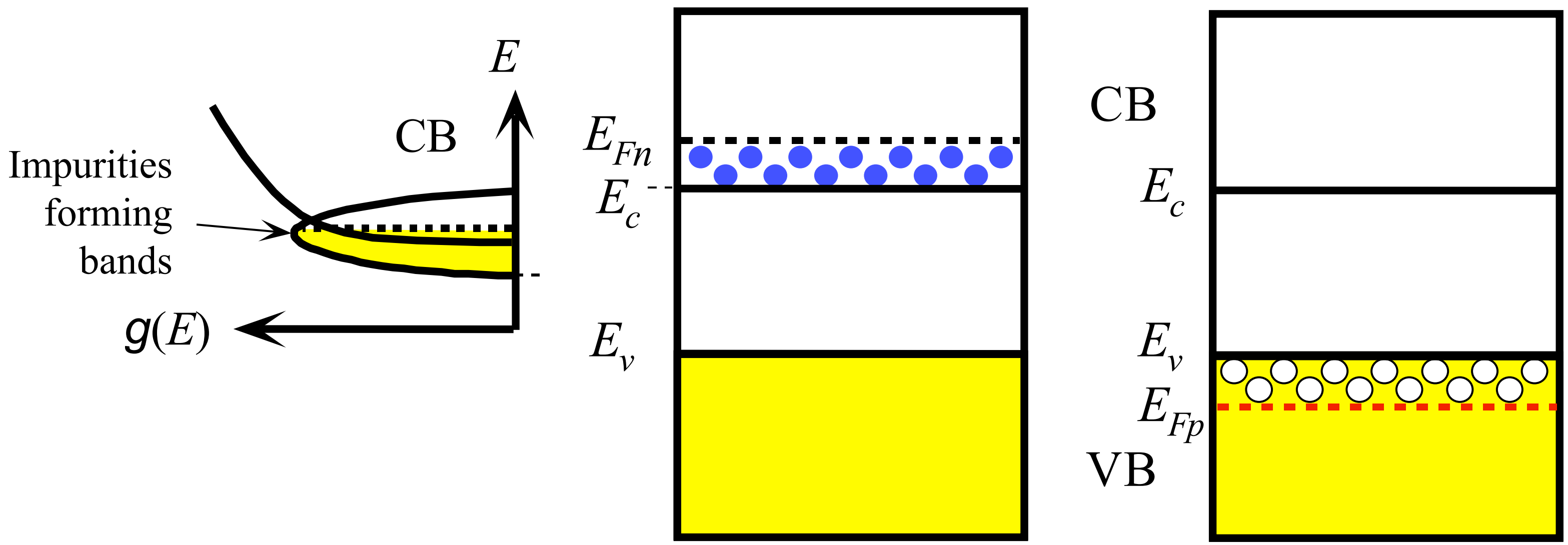
(a)



(b)

How do electrons and holes populate the bands?

- Heavily Doped Dopant States



Degenerated n-type semiconductor
Large number of donors form a band that overlaps the CB

Degenerated p-type semiconductor

Intrinsic semiconductor

n = p and therefore:

$$2\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{3/2} e^{(E_F - E_c)/k_B T} = 2\left[\frac{2\pi m_h^* k_B T}{h^2}\right]^{3/2} e^{(E_v - E_F)/k_B T}$$

Here we can have a position of Fermi level in intrinsic SC:

$$E_F(T) = \frac{E_g}{2} + \frac{3}{4} k_B T \ln\left[\frac{m_h^*}{m_e^*}\right]$$

So, it is almost in the middle of the bandgap

Extrinsic Semiconductors

* Charge-neutrality equation

$\Rightarrow$ Evaluation of carrier concentrations

* Fermi-level variation in extrinsic semiconductors

$\Rightarrow$ Doping dependence

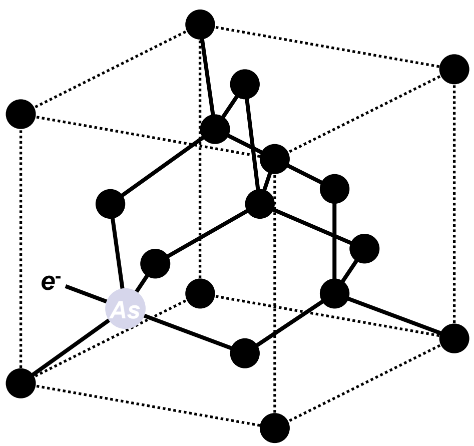
* Impurity statistics

Charge-Neutrality Equation

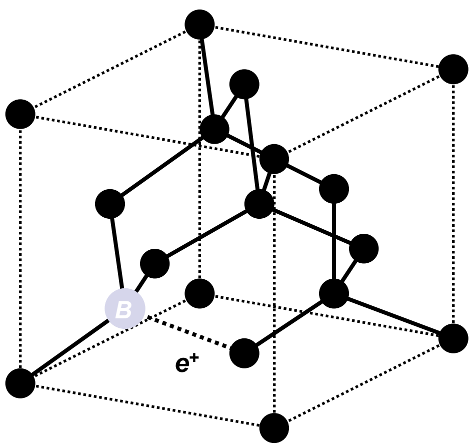
• Previously we discussed how we may improve the electrical properties of semiconductors by the deliberate addition of impurities known as DOPANTS

* The doped semiconductor is referred to as EXTRINSIC since at room temperature its electrical characteristics are determined by the concentration of DOPANTS rather than the intrinsic carrier concentration n_i

⇒ Doping with DONORS increases the ELECTRON concentration and the resulting semiconductor is said to be *n*-TYPE while doping with ACCEPTORS increases the HOLE concentration and the resulting semiconductor is said to be *p*-TYPE



REPLACING A SILICON ATOM WITH ARSENIC YIELDS AN EXTRA ELECTRON



REPLACING A SILICON ATOM WITH BORON YIELDS AN EXTRA HOLE

Charge-Neutrality Equation

• The carrier concentrations in extrinsic semiconductors may be determined by introducing a CHARGE-NEUTRALITY equation which expresses the fact that the crystal as a whole must remain CHARGE NEUTRAL

$$p - n + N_D^+ - N_A^- = 0$$

* In this expression N_D^+ and N_A^- are the concentrations of IONIZED donors and acceptors

⇒ When an electron is liberated from a donor it is left with an equal POSITIVE charge while liberation of a hole from an acceptor leaves it NEGATIVELY charge

* Another important equation that will be used to determine the carrier concentrations in the extrinsic semiconductor is

$$np = 4\left[\frac{2\pi k_B T}{h^2}\right]^3 (m_e^* m_h^*)^{3/2} \exp\left[-\frac{E_g}{k_B T}\right]$$

⇒ Since the RHS of this equation is INDEPENDENT of the doping the PRODUCT of the electron and hole concentrations in a non-degenerate semiconductor is similarly INDEPENDENT of the doping concentration

Fermi-Level Variation in Extrinsic Semiconductors

- **In doped semiconductors:**

$$n = n_i \exp[(E_F - E_i)/k_B T]$$

$$p = n_i \exp[(E_i - E_F)/k_B T]$$

* **By REARRANGING these equations we obtain expressions relating the Fermi level to electron and hole concentrations in doped semiconductors:**

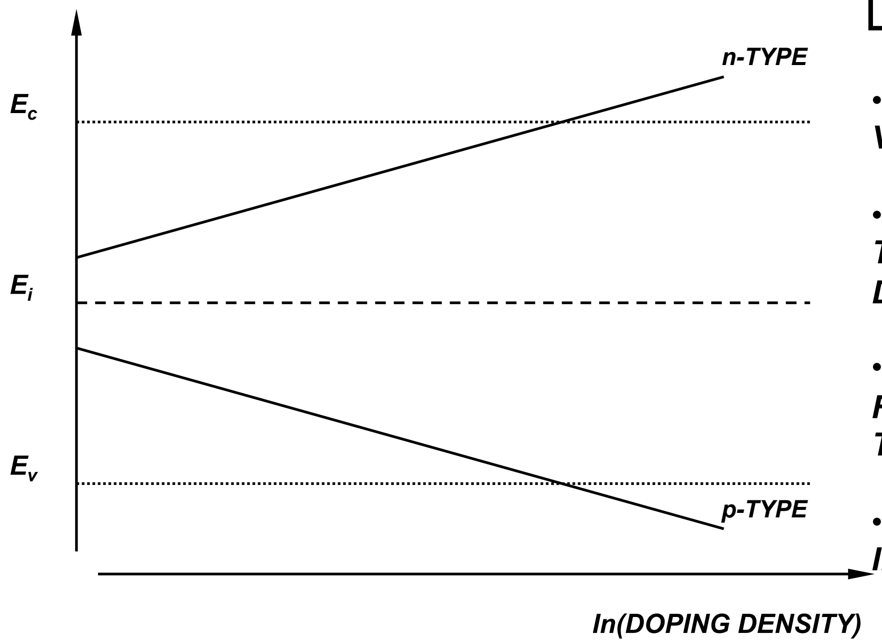
$$E_F - E_i = k_B T \ln\left[\frac{n}{n_i}\right] = -k_B T \ln\left[\frac{p}{n_i}\right]$$

Fermi-Level Variation in Extrinsic Semiconductors

• For **HEAVY** doping and **FULL** ionization of dopants the position of the Fermi energy in the gap varies as

$$E_F - E_i = k_B T \ln\left[\frac{N_D}{n_i}\right], \quad N_D > N_A >> n_i$$

$$E_F - E_i = -k_B T \ln\left[\frac{N_A}{n_i}\right], \quad N_A > N_D >> n_i$$



• *SCHEMATIC ILLUSTRATION INDICATING THE FERMI LEVEL VARIATION IN A SEMICONDUCTOR WITH n- AND p-TYPE DOPING*

• *THE DOPING RANGE ILLUSTRATED HERE CORRESPONDS TO THE REGIME OF HEAVY DOPING WHERE THE DOPANT DENSITIES SIGNIFICANTLY EXCEED n_i*

• *NOTE THAT FOR SUFFICIENTLY HIGH DOPING DENSITIES THE FERMI LEVEL MOVES CLOSE TO EITHER BAND EDGE SO THAT THE SEMICONDUCTOR BECOMES DEGENERATE*

• *THE TERM "DEGENERATE" IS THEREFORE OFTEN USED TO INDICATE A VERY-HEAVILY DOPED SEMICONDUCTOR*

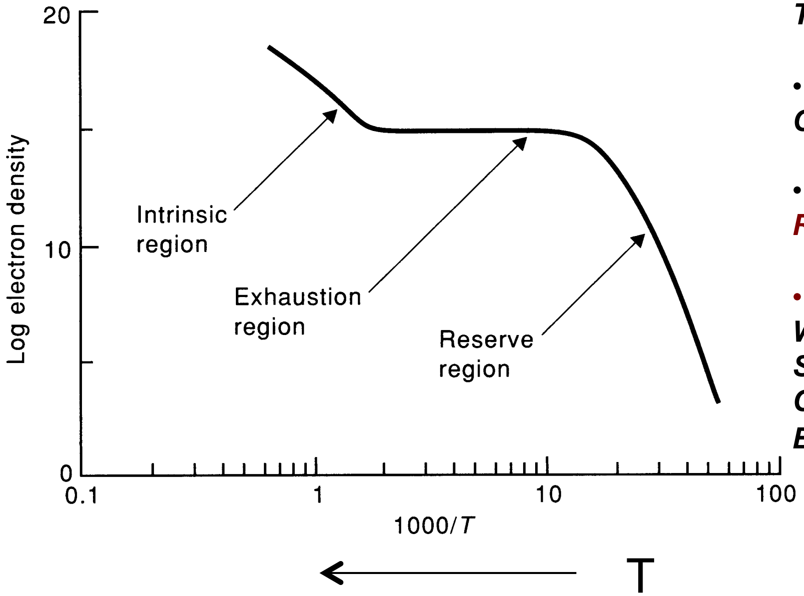
Impurity Statistics

- At **HIGH** temperatures $N_c >> N_D$ and

$$n = N_D$$

*** In this case the Fermi energy will lie well BELOW the donor energy as ALL the donors are ionized and the density is therefore relatively CONSTANT**

*** The variation of the electron density with temperature in a typical *n*-type semiconductor is shown schematically in the figure below**



- ***VARIATION OF THE ELECTRON DENSITY WITH TEMPERATURE IN A TYPICAL n-TYPE SEMICONDUCTOR***
- ***THE REGION WHERE THE DENSITY IS INDEPENDENT OF TEMPERATURE IS KNOWN AS THE EXHAUSTION REGION***
- ***AT LOWER TEMPERATURES THE DONORS BEGIN TO FREEZE-OUT CAUSING A REDUCTION OF THE ELECTRON CONCENTRATION***
- ***THIS LOW-TEMPERATURE REGION IS OFTEN REFERRED TO AS THE RESERVE REGION***
- ***AT HIGHER TEMPERATURES THAN WE HAVE CONSIDERED HERE WE MAY NO LONGER NEGLECT THE HOLE CONCENTRATION AND THE SEMICONDUCTOR REVERTS TO INTRINSIC-LIKE BEHAVIOR WHERE CARRIERS FROM THE BULK CRYSTAL SWAMP ELECTRONS PROVIDED BY THE DONORS***